Purpose

This document introduces the basic concepts of Statistical Process Control (SPC). It is not designed to teach the reader how to implement SPC, but rather to explain what SPC is and why it works. There are many excellent courses on SPC and anyone responsible for implementing or participating in an SPC program should take one of these courses.

Background concepts

Before we can talk about SPC, we need to understand some fundamentals of inspection and statistics.

Specification limits / nominals and tolerances

When engineers design a part, they know what characteristics are important, what values they should have, and how far the part can deviate from those values before it becomes unsuitable for use. Characteristics can be anything from the location of the center of a hole, to the diameter of the hole, to the roughness of a surface, to the viscosity of a liquid product. The desired value is called the nominal, and how far a part can deviate before becoming unsuitable is called the tolerance. Tolerances can be unilateral or bilateral.

Unilateral tolerances

Unilateral tolerances have a limit on one side of the nominal but not on the other. This happens when deviation to one side of nominal is impossible or is of no concern. For example, the thickness of an undercoating must be at least 1 millimeter thick, with a desired thickness of 2 millimeters. Thicker coating is fine.

Bilateral tolerances

Bilateral tolerances have a limit on each side of the nominal. A bracket that is supposed to be 3 inches long but can be .02 inches longer or .01 inches shorter without adversely affecting performance is an example of a part with bilateral tolerancing.

Specification limits

Specification limits, also called engineering limits, are calculated from nominal and tolerances. The upper specification limit, USL, is nominal plus the plus tolerance. The lower specification limit, LSL, is nominal minus the minus tolerance. In the bilateral tolerance example above, the bracket has USL = 3.02 inches and LSL = 2.99 inches.

Inspection

SPC is an extension of inspection. Inspection is measuring a part and comparing the measurement to the specification limits. SPC takes the measurement obtained in inspection and compares it to previous values to detect subtle changes in the manufacturing process before they become big problems. Inspection uses simple math to detect gross errors, SPC uses complex math to identify problems while they are still small.

What is variation

Every process has variation. The variation may be so small it is considered insignificant, but it is always there. For example, if you picked up a ream of paper and measured the length of each sheet with a ruler, you might conclude that each sheet is exactly 11 inches long and there is no variation. However, if you measured each sheet again but used a very precise instrument, you would find some sheets are 11.0003 inches long, some are 10.9998 inches long, and so on. In modern manufacturing, differences that were once considered insignificant are now unacceptably large.

Basic definition

Variation is a term used to describe the amount of difference among a group of measurements that would ideally be identical. The measurements could represent the sizes of different pieces off a production line, the size of one piece measured at several points along the length of the piece, or repeated measurements taken at a single point on a single part. The variations would be described as piece to piece variations, within piece variation, and measurement variation respectively. In practice, every measurement contains all three of these types of variation. The three types add up to give total variation.

How to measure it - Standard Deviation

The most common measure of variation is the Standard Deviation, usually indicated by the letter s or the Greek symbol Sigma, σ. The standard deviation is the average amount each measurement in a group differs from the average for that group. There are several formulae for calculating Standard Deviation, and any book or course that teaches SPC or statistics will explain each formula and tell when to use it. For now you do not need to know a formula, you just need the concept, so here is a “slightly wrong” but easy to understand calculation. Find the average measurement by adding all measurements and dividing by the number of measurements. Find the deviation for each measurement by subtracting the average of all readings from that particular reading, and discarding the minus sign if the result is negative (if the average is 5.0, a reading of 4.8 has a deviation of 0.2 and a reading of 5.1 has a deviation of 0.1). Calculate the average deviation by adding up the deviations for each individual measurement and dividing by the number of measurements. As mentioned previously, this is not a correct formula for Standard Deviation (it consistently gives values that are too low), but it clearly shows the concept. As you can see, measurements that are “all over the place” will have a high standard deviation, and measurements that are very close together will have a small standard deviation. Standard deviation is never negative, and will only be zero if all measurements are identical.

Some related terms

When discussing variation, some related terms are often used. These terms are not used in SPC directly, but relating them to the concept of variation can help put variation in perspective.

Resolution

Resolution applies to measuring devices. Resolution is the smallest increment the device can display. For example, a digital speedometer that shows 1 digit to the right of the decimal place has a resolution of .1 miles per hour, and a tape measure that has 16 lines to the inch has a resolution of 1/16 of an inch. The smaller the interval, the “finer” or “higher” the resolution is.

Precision

The term Precision can be applied to either a process or an measuring device. A precise process yields product that has little variation, and a precise measuring device give readings that have little variation (if all of the readings come from the exact same point on the exact same part). In conversation “Precision” can be a somewhat vague term because it implies finer resolution as well as lower variation. This “conversational” definition is not inconsistent with the stricter definition (there is no point to displaying 6 digits if the last 3 are all over the scale) but it can be a source of misunderstanding.

Accuracy

Accuracy is the ability of a device to produce parts or measurements that have an average close to the desired average. For a production machine the desired average is nominal, for an inspection machine the desired average is the true value (as determined by a “master” gage, with calibration traceable to the national standards lab).

Distributions

A distribution is a way of describing the relative likelihood of different possible outcomes. For example, if you roll a die, you are as likely to roll a 6 as you are to roll a 1. All 6 outcomes have equal probability, 1 in 6. The distribution that describes rolling a single die is called the “uniform distribution”. If you plotted the uniform distribution on graph paper it would be a rectangle. Compare rolling 1 die to rolling 2 dice and adding the result. When you roll two dice, the chance of rolling a 7 is six times as likely as rolling a 2. If you plotted the distribution of two dice added, it would be a triangle. The distribution used most in SPC is the “Gausian” or “normal” distribution. The Gausian distribution is bell shaped. The Gausian distribution describes the combined effect of a very large number of unrelated and individually insignificant events. This is exactly what you have in an ideal production or measurement situation - the vibration of individual molecules because of heat, slight air currents, the gravity fluctuations of passing comets - all trivial and the total variation is small, but they can’t be removed entirely and they do produce an effect that is measurable if you look hard enough.

Random (inherent) variation vs. assignable causes

As we mentioned earlier, every process has some variation. Sometimes there is a reason for some of the variation. A loose bolt on the machine, worn gears, a dull tool, and many other causes can all contribute to the variation in a process. If you fixed all of those problems, there would still be variation in the process, but it would be much less. The amount of variation left after you fix all of the problems is called the inherent variation. The problems you fixed are called assignable causes.

Populations and Sampling

A population is any complete collection of individuals that have something in common. For example, “all registered voters” is a population, “all ¼ 20 screws made at Acme Screw Company” is a population, and “All people who read this document” is a population. An individual can be a member of more than one population, i.e. a registered voter reads this document. The concept of populations is useful because it allows you to give the scope or extent of a statement or question. The question “what percent of the population supports a balanced national budget” is much more relevant when the population in question is “all registered voters” than when the population is “all 8^{th} grade students in California”. In SPC, and in statistics in general, the term population is very common because you must specify exactly what you are collecting information on.

A sample is a subset of a population that is assumed to represent the population. Asking every registered voter for an opinion on a balanced budget is too expensive, but asking a few hundred taken from different demographic groups can give a reasonable estimate of the percentage of all voters who approve. In general, the bigger the sample the more it costs, but the more reliable the estimate is.

What is SPC

SPC is a mathematical tool for detecting changes in a manufacturing process. SPC is necessary part of an effective quality assurance program. Innovation or refinement provide improvements in quality, and SPC holds those gains, preventing backsliding. SPC makes the assumption that a process is running the same as it always has (since the last time you made an intentional change) and asks the question “if nothing changed, how likely is it that the process would have produced these results?”. If the answer is “very unlikely” the SPC chart or program informs the operator that there is a problem, and the operator can find the problem and fix it. Just like a micrometer can detect changes in size too small for the unaided eye, SPC can detect changes in a process too small for the unaided brain. SPC can be divided into two categories - Control Charts and Descriptive Statistics.

Variable data vs. attribute data

Data used for SPC can be divided into two types, variable and attribute. Variable data is measured on continuous scales. On continuous scales you can always pick a value between any two other values. If one part is 3.131 inches long and another is 3.132, you could pick a value of 3.1315 that is halfway between. Length, temperature, weight, time, and voltage are examples of continuous scales.

Attribute data is measured on discrete scales. Discrete scales are scales where you can pick two adjacent values and there are no values between them. The number of people in a room and the number of heads in 3 tosses of a coin are examples of discrete data. True/false and count measurements like “number of scratches per part”, “number of broken filaments”, and “is the part cracked” are all attribute measurements.

Control charts

Control charts are the “dashboard warning lights” for a manufacturing process. They are an immediate warning that something is wrong. There are many styles of control charts, but the most popular ones involve taking a small random sample of product, called a subgroup, at fixed intervals. The product is measured and the results are tabulated, calculated, and plotted. The charts all have control limits and any point that falls outside the control limits signals a problem. Optionally, certain patterns can also signal a problem. To get the maximum benefit, minimize the time between producing the parts and plotting the subgroup - if there is a problem the longer it takes to find out about it, the more bad parts are produced! In practice, two control charts are used simultaneously. A chart of averages, medians, or individuals is used to detect subgroup to subgroup drift, and a range or sigma chart is used to detect within subgroup drift. The choices of which charts to use, what subgroup size to use, and how often subgroups should be taken are beyond the scope of this document. The calculation of control limits is also beyond the scope of this document but is unnecessary because QC-CALC calculates them for you.

XBar control charts

XBar, sometimes written as an X with a solid line above it, is statistical notation for the average. An XBar chart is a plot of the subgroup averages. The XBar control chart is the most popular control chart. It is used to detect variation between subgroups.

Range control charts

The range chart plots the difference between the largest and smallest readings in each subgroup. It is easier to calculate than the sigma control chart but is not quite as sensitive. The range chart detects within subgroup variation.

Sigma (standard deviation) control charts

The sigma control chart plots the standard deviation of each subgroup. Like the range chart, it detects within subgroup variation. When control chart calculation and plotting was done by hand the range chart was strongly preferred over the sigma chart but now that computers perform the calculations the range chart is losing ground to the sigma chart.

Other control charts

Most charting is done on XBar and Range or XBar and Sigma chart pairs. Some special cases call for other charts. QC-CALC also provides Moving Average, Moving Range, Individuals, and Median control charts.

Moving average

The moving average chart uses overlapping subgroups. If 6 measurements are collected and the subgroup size is 3, a conventional average (XBar) chart would have 2 points (parts 1-3 and parts 4-6), but a moving average would have 4 (1-3, 2-4, 3-5, and 4-6).

Moving range

Like moving average but the range is calculated on overlapping subgroups.

Individuals

Subgroup to subgroup drift is ignored.

Medians

Subgroup to subgroup drift is measured by the median of the subgroup not the average. To calculate the median arrange all the points in the subgroup from smallest to largest. If the number of points in the subgroup is odd the median is the middle point. If the number of points in the subgroup is even the median is the average of the two middle points.

And more all the time

There are many other charts in use throughout industry and all have supporters. Refer to a detailed SPC book or course for a current list, and information about when each is appropriate.

Descriptive statistics

If control charts are described as the “dashboard warning lights” for a manufacturing process, descriptive statistics would be an advanced diagnostic computer. While control charts are typically used by the operator producing the parts, descriptive statistics are usually used by a QC Engineer long after the parts are produced.

Process capability (Cp, Cpk, Ppk)

Process capability compares the variation in the process to the specification limits (nominals and tolerances). Even though none of the parts are bad, a process capability study can extrapolate and predict how many parts per million will be out of spec. A process capability study can also compare parts made under different conditions - for example if two machines are making the same part a process capability study can tell which one is better in the long run.

Histogram

Histogram plots look at the data grouped by size. They are used to detect some patterns that are impossible to detect in time oriented data. For example, if 5% of the parts in a barrel are supposed to be in a different barrel because they are a slightly different size, a histogram will often show that.

Pareto

When multiple features on a part are measured, a Pareto chart sorts them from worst to best so you can concentrate your efforts on the biggest problems.

Correlation and regression

Looks for relationships in the data. Are the holes punched in a part smaller when the metal is thicker? Does a higher oven temperature correspond to less resistance in solder joints? Does surface finish or post length affect insertion force? Correlation and regression analysis can help answer questions like these.

Gage repeatability and reproducibility

A Gage R&R study is a method of determining the suitability of a gage or gaging system for measuring a particular process. Every measurement taken has some error associated with it, and if this error is large compared to the allowable range of values (the tolerance band), the measuring device will frequently accept bad parts and reject good ones. A Gage R&R study partitions the variation in a group of measurements and says what variation comes from the parts being measured, what variation comes from operator differences (reproducibility), and what variation comes from the gage itself (repeatability).

Tests of normality

Some calculations assume the data is distributed according to a “Gausian” or “normal” distribution. QC-CALC provides several tests to determine if the data is “normal” and therefore can be used in the calculations. The tests available are Chi squared, Skewness, Kurtosis, and Probability plot.